are given in Eqs. (12) and (13), respectively. It can be seen that the expressions for the shearing stress-resultants satisfy the following conditions: at the point where the torque is applied the stress-resultant is maximum, and it vanishes at infinity.

The results obtained in Eqs. (12) and (13) may be used to verify that the total torque produced by the force-resultants is equal to the applied torque. The calculation of the total torque can be made by summing up all the force-resultants multiplied by their corresponding moment arms of an infinitesimal element in the immediate vicinity of the applied torque. The normal stress-resultants acting on the element in both x and s directions are obtained from the equilibrium condition in terms of the corresponding shearing stress-resultants. The result proved to be correct.

Figures 2 and 3 show the shearing stress-resultants distribution along the generatrix (between $x = h$ and $x = 500h$) of an infinitely long cylinder with $a/h = 100$. The computation of the shearing stess-resultants was made with the aid of a CDC 1604 digital computer. It was noted that the shearing stress-resultant near the origin due to the applied torque composed of a pair of forces in the x direction is much larger than that due to a pair of forces in the s direction. The reason for the discrepancy is due to the effect of the curvature of the shell on the displacements. The results also show that the shearing stress-resultants decrease very rapidly from the region near the applied torque outward along the generatrix.

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Natural Frequency Curves of Simply Supported Cylindrical Shells

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THE problem of determining the natural frequency of cylindrical shells has been studied by many authors. A brief historical review may be found in Ref. 1. So far, the frequency curves obtained by the former investigators do not cover the ranges of wave number and shell radius-thickness ratio broad enough for practical use.

In this brief note, a set of frequency curves is presented. They cover the radius-thickness ratio from 100 to 1500, the circumferential wave number from 0 to 38, and axial wave number ma π/l from 0.3 to 30.

The equation of motion used was obtained in the manner of Morley[2]:

$$\nabla^4(\nabla^2 + 1)^2 w + 4K^4 \frac{\partial^4 w}{\partial x^4} + \frac{a^4}{D}\rho h \nabla^4 \frac{\partial^2 w}{\partial t^2} = 0 \qquad (1)$$

Here w is the quantity of the radial displacement w' divided by the shell radius a. x' and θ are the cylindrical coordinate system and $x = x'/a$:

$$\nabla^2 = (\partial^2/\partial x^2) + (\partial^2/\partial\theta^2)$$

$$D = Eh^3/12(1 - \nu^2)$$

$$K^4 = 3(1 - \nu^2)(a/h)^2$$

E and ν are, respectively, Young's modulus and Poisson's ratio, and ρ and h are, respectively, the mass density of the material and the thickness of the shell.

For a simply supported cylindrical shell in free vibration, the radial motion is taken as

$$w = e^{i\omega t} \times C_{sn} \cos sx \times \cos n\theta \qquad (2)$$

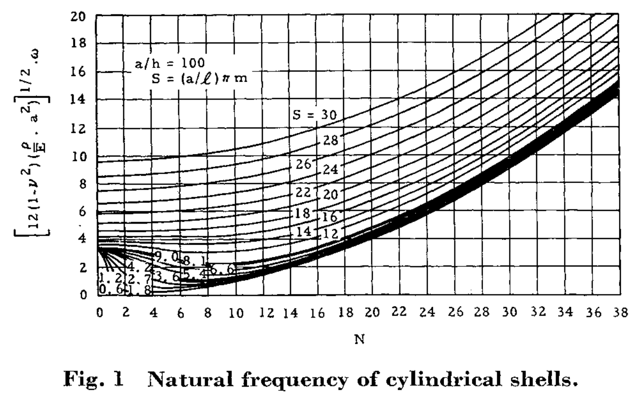

Fig. 1 Natural frequency of cylindrical shells.

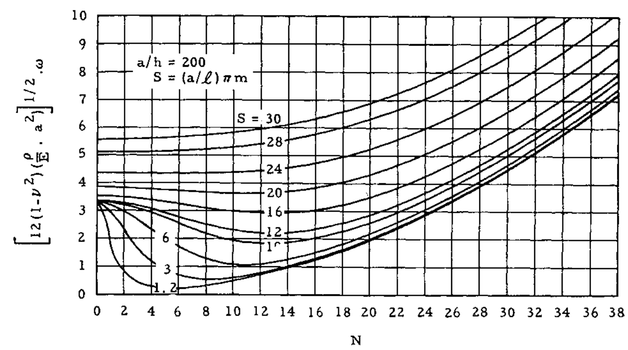

Fig. 2 Natural frequency of cylindrical shells.

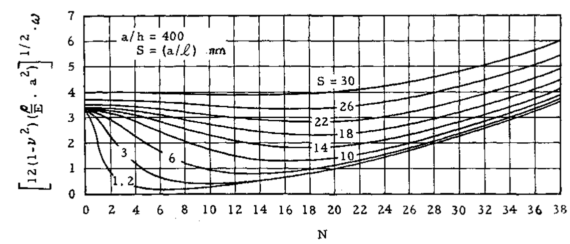

Fig. 3 Natural frequency of cylindrical shell.

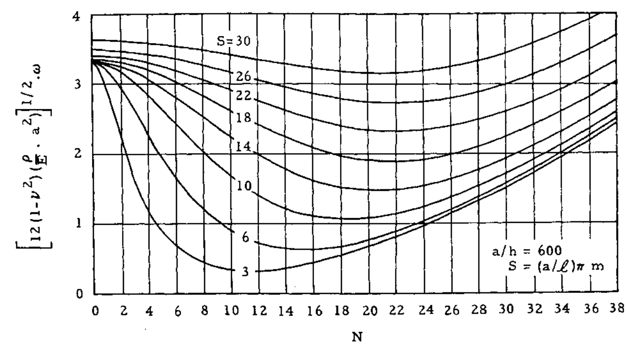

Fig. 4 Natural frequency of cylindrical shell.

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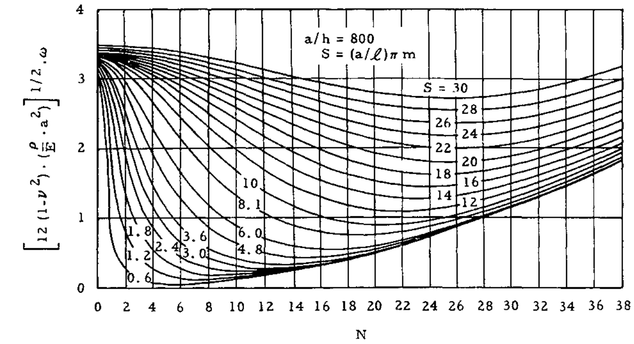

Fig. 5 Natural frequency of cylindrical shells.

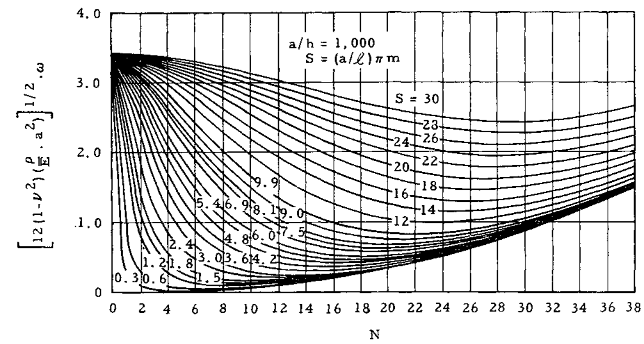

Fig. 6 Natural frequency of cylindrical shells.

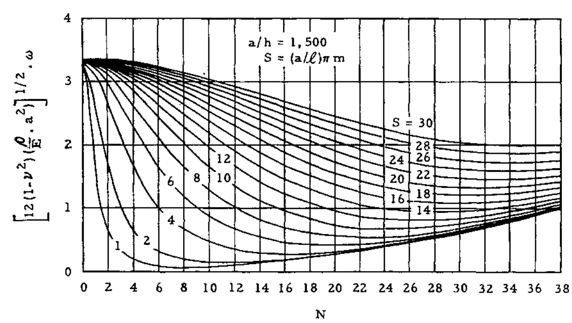

Fig. 7 Natural frequency of cylindrical shells.

where $s = ma\pi/l$. The cylinder length is l, and m is the number of half-waves in the x direction. Combining Eqs. (1) and (2) yields the following frequency equation:

$$\left\{\left[12(1-\nu^2)\cdot\left(\frac{\rho}{E}\right)\right]\cdot a^2\right\}^{1/2}\cdot\omega_{sn} = \frac{h/a}{s^2+n^2}\times [(s^2+n^2)^2(s^2+n^2-1)^2+4K^4s^4]^{1/2} \quad (3)$$

Charts based on Eq. (3) are prepared with

$$\left[12(1-\nu^2)\,\frac{\rho}{E}\,a^2\right]^{1/2}\omega_{sn}$$

as the frequency parameter; shown in Figs. 1–7. It is believed that the range of the curves thus covered is broad enough for engineering uses.

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Separated Flow Behind a Rearward-Facing Step with and without Combustion

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Introduction

Reduction of the transonic and low supersonic base drag of aircraft designed to operate efficiently at hypersonic speeds has become an important goal in recent years. Base drag is an important component of the total drag of such aircraft because, typically, optimum hypersonic bodies have slender, pointed forebodies and blunt afterbodies. One scheme that has been proposed as a means for reducing the base drag of hypersonic vehicles traveling at transonic and moderate supersonic speeds is to inject fuel into the region of separated flow at the base and to burn this fuel in the free boundary layer separating the base flow from the main stream. The experimental program reported here was devised to allow the study of a closely related problem, viz., the problem of the separated flow field behind a rearward-facing step with and without hydrogen combustion in the separated region. Experimental results are compared with corresponding predictions of an approximate analytic model.

Description of the Model and Its Instrumentation

A simplified vertical center plane sectional drawing of the model is presented in Fig. 1. A two-dimensional, convergent-divergent half-nozzle was designed by the method of characteristics to provide a uniform, supersonic ($M \approx 1.5$) air stream to the upstream side of a rearward-facing step. This half-nozzle has a constant width of 6 in. The height of the nozzle entry section is 3 in., of the throat, 1 in., and of the exit, 1.23 in. (including an allowance for the development of a turbulent boundary layer downstream of the throat of the half-nozzle). The upper surface of the half-nozzle is contoured; the lower surface is plane. A $\frac{1}{2}$-in.-deep by 6-in.-wide rearward-facing step is formed by a $\frac{1}{2}$-in. offset of the duct wall at the exit plane and on the contoured (upper) side of the half-nozzle. A $\frac{1}{16}$-in.-high slot spans the rearward-facing plane surface (riser) of the step at a distance of $\frac{1}{16}$in. from the outside corner of the step. This slot allows hydrogen (from the 4-in. central section of the plenum) and cooling air fo- the side glasses (from the two 1-in. side sections of the plenum) to be injected in a direction that is, as nearly as possible, parallel to the direction of and contiguous to the main air flow at the nozzle exit. The side walls of the nozzle are steel to a station located about $\frac{1}{2}$in. upstream from the nozzle exit. Plate glass windows, 1in. thick, extend the side walls from this station to a station about 9 in. further downstream. These glass side

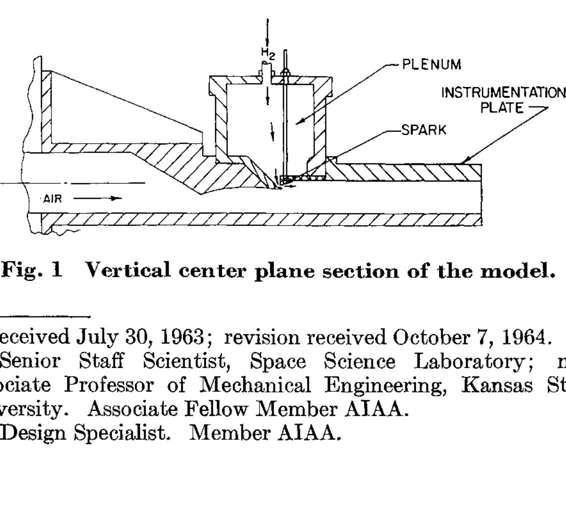

Fig. 1 Vertical center plane section of the model.

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